Is It Absurd to Deny Bivalence?

(or: 'It's vagueness all the way down…')

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I. Introduction

In cases of borderline vagueness, it seems intuitive to characterise certain propositions as *neither true nor false* on the basis of it being unclear which, if either, of these truth values obtain. In practice, a typical language user might simply refrain from using terms like 'bald', 'heap', 'thin', etc. in such situations, for example when referring to a man who is partially bald, or prefix them with a suitable qualifier such as 'partially', 'almost' or 'slightly'. However, it is unclear how we should treat such expressions in formal logic, as according to the principle of bivalence, all well-formed sentences must possess one of two possible truth values: true and false. Furthermore, it is widely held that our grasp upon the meaning or 'sense' of a sentence depends upon its truth conditions. This semantic conception of truth is represented by Tarski's *Convention T* (Gómez-Torrente 2006), which may be symbolised as follows:

(T1) $T[\text{'P'}] \leftrightarrow P$

(T2) $T[\text{'}{\sim}P\text{'}] \leftrightarrow {\sim}P$

where T[] signifies the metalinguistic truth predicate. According to T1, the sentence 'Roses are red' is true if and only if roses actually are red. But what if roses are not red, but a borderline case of both red and pink? In this case, it is not true that roses are red, but neither is it clearly false. Similarly, the negation of this sentence ('It is not the case that roses are red') would, under the same conditions, seem to lack a determinate truth value. In order to cater for the use of vague predicates in formal logic, it seems that we must adopt a third, or 'indeterminate', truth value, which amounts to a denial of the principle of bivalence.

II. Williamson's *Reductio* Argument

In his seminal 1992 paper, 'Vagueness and Ignorance', Timothy Williamson presents a *reductio ad absurdum* argument against the denial of bivalence on the basis that — in cases of borderline vagueness, at least — it amounts to a denial of the Law of the Excluded Middle (LEM) in the metalanguage, i.e:

(W1) ${\sim}(T[\text{'P'}] \vee T[\text{'}{\sim}P\text{'}])$

By substituting the two disjuncts for the right-hand side of the biconditionals in T1 and T2, this gives:

(W2) ${\sim}(P \vee {\sim}P)$

which, by De Morgan's Law, entails the contradiction:

(W3) ~P & ~~P

This is clearly unacceptable to any proponent of a multivalent logic who wishes to deny bivalence. The source of this problem may be traced back to Williamson's use of a classical (i.e. bivalent) logic in the metalanguage of W1 to force a contradiction in the object language by application of Tarski's *Convention T*. In effect, the classical assumptions of the metalanguage 'leak through' into the object language, reducing the denial of bivalence to absurdity. To refute this argument, then, the multivalentist must either adopt a non-classical metalanguage, or reject T1 and T2 — the basis of the semantic conception of truth.

III. Multivalent Logics and Assertibility Conditions

So, is the multivalentist committed to Williamson's assumptions? Pelletier and Stainton (2003: 372) deny that one need accept W1 provided that the negation operator always yields either true or false, even in indeterminate cases.[1] On this view, LEM holds because the second disjunct of W1 comes out as true even when P is neither true nor false, and so Williamson's argument never even gets off the ground. Furthermore, they argue, it is unclear whether the multivalentist should accept both directions of the Tarskian biconditionals. Although the left-to-right direction (T['P'] → P) is uncontentious, on the basis that *something* must make it true that P, the reverse (P → T['P']) does not necessarily apply in a three-valued logic. If P is indeterminate, then it is precisely *not* the case that P, and so the right-to-left direction of the biconditional fails (*ibid.* 375). This depends upon the logic of the conditional operator in cases where the antecedent is indeterminate, but regardless of whether such statements are taken to be false or indeterminate (the two available options), T1 and T2 will fail to hold in all cases under the resulting multivalent logic. On this account, T1 and T2 '*already presuppose bivalence*, and so they are not the appropriate disquotational schemas for a multivalued logic' (*ibid.* 374).

Williamson's objection to this line of argument is that without the complete disquotational schema (i.e. one that exhaustively specifies whether every sentence in our object language is true or false) that is provided by the Tarskian biconditionals, we lose our grip on the meaning or 'sense' of the sentence (Williamson *op. cit.* 268). Given that most concepts contain at least some element of vagueness, this effectively renders vast tracts of our language meaningless, which constitutes a further argument against the denial of bivalence, or in favour of nihilism (cf. Unger 1979). However, this claim is too strong. In order to grasp the meaning of a proposition, it is arguably sufficient that we know the cases in which it is *determinately* true or *determinately* false. In cases of borderline vagueness, where it is unclear that proposition or its negation can be asserted, the use of such a proposition would simply be

[1] They call this 'exclusion negation' on the basis that only one particular truth value is being denied, or 'excluded'.

incorrect or *inappropriate* rather than strictly *meaningless*.[2] On this account, the meaning of a sentence is still determined by its (determinate) truth conditions, but its *use* is determined by a separate set of assertability conditions. In the case of vague predicates, these two sets of conditions come apart, leaving 'gaps' in our assignment of determinate truth and falsity. This results in what Pelletier and Stainton (*op. cit.* 371) call a 'truth-value gap' theory, and may be captured in logical form using the following biconditionals:

(T1*) T['P'] ↔ ΔP
(T2*) T['~P'] ↔ Δ~P

where Δ is a new operator signifying 'It is determinate that'.[3] Substituting T1* and T2* into Williamson's original argument, we obtain the highly intuitive and non-contradictory conclusion:

(W3*) ~ΔP & ~Δ~P

i.e. 'it is not the case that *determinately* P, nor is the case that *determinately* not-P' — which is precisely what the opponent of bivalence would expect to say in cases of borderline vagueness.

IV. Higher-Order Vagueness

The introduction of the 'determinately' operator, however, presents another problem: at what point does a proposition become determinately true or determinately false? Although we can cite clear cases of determinate truth and falsity (e.g. 'snow is white', 'all swans are white', etc.), it is unclear exactly where the boundaries lie. In other words, *determinate truth is itself a vague notion*, thus giving rise to the problem of 'higher-order vagueness' (Williamson 2003: 8). Indeed, as Williamson points out, any logic that cannot account for the existence of higher-order vagueness is counterintuitive (although perhaps not absurd), as we cannot deny that the concept of vagueness itself permits of borderline cases (*ibid.*). In order for the multivalentist denial of Williamson's *reductio* to hold water, it must therefore also be able to give an account of higher-order vagueness.

There are two ways in which this might be done. The first is to allow that the truth value of the determinately operator may itself be determinate. Thus the statement ΔP may be neither true nor false in cases of higher-order vagueness (although ΔΔP or ΔΔΔP may be determinate, as multiple applications of the operator correspond to ever increasing orders of vagueness). This is, however, unattractive, as it would make the determinately operator non-truth functional. The second is a variation of a proposal by McGee and McLaughlin (1995: 229–30), which accounts for higher-order vagueness in terms of correspondingly higher-orders of metalanguage. On this account, the

[2] Note that we can also use sentences 'incorrectly' as a form of exaggeration, or to make a point, even though the resulting utterance may be literally false.

[3] I have used the term 'determinate' in preference to 'definite' in order to avoid begging the question of whether vagueness is an epistemic, metaphysical or linguistic issue.

‘determinately’ operator becomes a function of the metalanguage (comparable to T[]) by which the semantics of the object language are defined. Accordingly, the logic of first-order vagueness is determined by a set of constraints upon the correct use of object language statements (i.e. their assertability conditions) along with a series of modified Tarskian biconditionals, such as:

(T1**) $\Delta T[\text{‘}P\text{’}] \leftrightarrow \Delta P$ (*ibid.*)

Second-level vagueness may then be described by a set of constraints upon the *metalanguage*, with a higher-order metalanguage (the metametalanguage) governing the formal semantics of metalinguistic terms, such as the meaning of ‘determinately’. This pattern may be repeated indefinitely to cope with any number of orders of vagueness without fear of generating a contradiction — or, as McGee and McLaughlin put it, ‘[a]s we ascend the hierarchy of metalanguages, we find vagueness all the way up’ (*ibid.* 230).

V. Conclusion

Williamson’s *reductio* represents a forceful attack on the denial of bivalence. However, its assumption that the metalanguage of a multivalent logic is necessarily classical is clearly question begging and, when combined with the appropriate logical constants and an account of higher-order vagueness, as described above, the alleged absurdity gives way to mere complexity. From this we can conclude that although it is not absurd to deny bivalence, such a move takes us far from the simplicity and elegance of classical logic, as well as requiring modifications to Tarski’s semantic conception of truth, as embodied by *Convention T*, which is unsuitable for dealing with the logic of vagueness. Whether this in itself constitutes an argument for retaining bivalence, as Williamson (1992: 279–80) claims, is open to question, although there are obvious advantages to employing a classical approach in cases where vagueness is not an issue, or can safely be ignored.

Perhaps a more promising approach to the problem of vagueness, however, lies in moving away from the truth conditional semantic theory of meaning altogether, and towards a notion of truth and falsity that is based on assertibility conditions along with paradigm cases of determinate truth and falsity. As Williamson (1992: 266 fn.) himself notes, this does not require a *denial* of bivalence, but rather that we *refrain from asserting it* in certain cases, such as those of borderline vagueness. Whilst much work remains to be done in order to flesh out the details of such an account, it might just offer the elusive possibility of combining the simple elegance of classical logic with a more complex, and arguably more sophisticated, notion of truth and meaning.

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