Is It Absurd to Deny Bivalence?

(or: 'It's vagueness all the way down…')

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Terminology

Object-language: the language that is being formally described (e.g. classical logic)
Metalanguage: the language used to state the properties of the object-language (e.g. English)
Principle of Bivalence: every statement is either true or false (the only two truth values)
Law of the Excluded Middle (LEM): $P \lor \sim P$ is true for all values of P

Supervaluationism: statements are true or false relative to a given 'sharpening' (Fine).
Epistemicism: statements have exact meanings that supervene upon their use, but we cannot track these meanings precisely, giving rise to epistemic vagueness (Williamson).

The Sorites (Heap) Paradox

(S1) One grain of sand is not a heap.
(S2) If *n* grains of sand is not a heap, then adding another grain does not make it a heap.
(S3) *(by repeated applications of modus ponens)* 1,000,000 grains of sand is not a heap.

Williamson's *Reductio* Argument

(W1)	~(T['P'] ∨ T['~P'])	Denial of LEM in the metalanguage
(T1)	T['P'] ↔ P	Tarski's *Convention T*
(T2)	T['~P'] ↔ ~P	
(W2)	~(P ∨ ~P)	From W1, T1 and T2
(W3)	~P & ~~P	By De Morgan's Law: ~(A ∨ B) ⊢ ~A & ~B

Exclusion Negation

P	~P
T	F
I	T
F	T

Semi-Standard Biconditional

P	T['P']	T['P'] → P	P → T['P']
T	T	T	T
I	F	T	F or I?
F	F	T	T

The Determinately Operator (Δ)

(T1*) T['P'] ↔ ΔP
(T2*) T['~P'] ↔ Δ~P
(W3*) ~ΔP & ~Δ~P

Higher-Order Vagueness (the vagueness of vagueness)

(a) ΔP (first-order)
ΔΔP (second-order)
ΔΔΔP (third-order) …

(b) (T1**) ΔT['P'] ↔ ΔP
(T2**) ΔT['~P'] ↔ Δ~P
(McGee & McLaughlin 1995)

A copy of the full paper, including references, may be downloaded from http://web.mac.com/keith.wilson/philosophy/logic/bivalence.pdf — e-mail: kw503@york.ac.uk